On some divisibility properties of binary recurrences

Florian Luca

Wits and UNAM

`florian.luca@wits.ac.za`

Coauthors: Yu. F. Bilu, T. Komatsu, A. Pizarro, P. Stănică

Let a, b be integers with $a^2 + 4b \neq 0$ and let $(U_n)_{n \geq 0}$ be the Lucas sequence given by $U_0 = 0$, $U_1 = 1$ and $U_{n+2} = U_{n+1} + U_n$ for all $n \geq 0$. In the first part of my talk, I will survey some divisibility properties of such sequences. In the second part of my talk, I will show that if $b \in \{\pm 1\}$ and (k, m, n, s) are positive integers such that

$$U_m \mid U_{n+k}^s - U_n^s,$$

and s is minimal such that the above divisibility holds (when k, m, n are fixed), then either $s \in \{1, 2, 4\}$, or $m < 20000(sk)^2$. The proof relies on solving certain Diophantine equations involving roots of unity.