SEMINARIO ARITMÉTICA Y GEOMETRÍA EN VALPARAÍSO

Fecha: Miércoles 10 de mayo, 2017
Hora: 14:00-15:30
Lugar: IMA PUCV, Sala 2-2

Antonio Laface (Universidad de Concepción)

Title: On blowing up the weighted projective plane

Abstract: Given three pairwise coprime positive integers a, b, c we denote by

$$\mathbb{P}(a,b,c) := \mathbb{C}^3 \setminus \{(0,0,0)\}/\sim$$

the corresponding weighted projective plane, where $(x,y,z) \sim (\lambda^a x, \lambda^b y, \lambda^c z)$ for any $\lambda \in \mathbb{C}^*$. The zero set of a homogeneous polynomial $f \in \mathbb{C}[x,y,z]$, with respect to the grading $\deg(x) = a$, $\deg(y) = b$, $\deg(z) = c$, is a well defined curve in $\mathbb{P}(a,b,c)$ whose multiplicity at $[1:1:1] \in \mathbb{P}(a,b,c)$ is the *multiplicity* of f. Denote by $V_{(d,-m)}$ the vector space of degree d homogeneous polynomials of multiplicity at least m and form the $\mathbb{Z}^2$-graded (Cox) ring [1]

$$R = \bigoplus_{(d,-m)\in\mathbb{Z}^2} V_{(d,-m)} t^{-m} \subseteq \mathbb{C}[x,y,z,t^{\pm 1}].$$

It is an open problem in general to decide if R is a finitely generated $\mathbb{C}$-algebra [1–4, 6]. This problem is related to the existence of irrational Seshadri constants [7, Remark 5.1.13] and to the Nagata conjecture [7, Remark 5.1.13]. In the first part of the talk I will discuss a necessary and sufficient condition for finite generation, while in the second part I will show for which values of a, b, c the ring R is generated by homogeneous elements of multiplicity at most 2.

This is joint work with J. Hausen and S. Keicher [5].

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References

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[2] Ana-Maria Castravet and Jenia Tevelev, $\overline{M}_{0,n}$ *is not a Mori dream space*, Duke Math. J. **164** (2015), no. 8, 1641–1667.

[3] José Luis González and Kalle Karu, *Some non-finitely generated Cox rings*, Compos. Math. **152** (2016), no. 5, 984–996.

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[5] ———, *On blowing up the weighted projective plane*, arXiv:1608.04542.

[6] Yi Hu and Sean Keel, *Mori dream spaces and GIT*, Michigan Math. J. **48** (2000), 331–348.

[7] Robert Lazarsfeld, *Positivity in algebraic geometry. I*, Vol. 48, Springer-Verlag, Berlin, 2004. Classical setting: line bundles and linear series.

http://seminarioaritmeticaygeometria.wordpress.com

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